In chapter 6 we discussed the free electron model. This model assumes that one or more of the outer electrons of the atoms can move freely through the crystal, so is no longer bounded to the atom. So the motion of those free electrons is not influenced by the positive ion cores of the atoms in the lattice. Their only energy is the kinetic energy which is:

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$
[1]

Materials well described by a free electron mode are the alkali metals, i.e. lithium, sodium, potassium, cesium and rubidium. Their atoms have a single electron in the outer shell. For a solid of such atom, this outer electron, valence electron, can easily break from the ion core, and freely move through the crystal. Kittel mentions Na and emphasizes that for a Na-solid most of the crystal consists of free space. The Na^+ ions have a radius of 0.98A, whereas half of the nearest-neibhbor distance of the metal is 1.83 A.

The free electron model was used to explain Ohm's law and the relation between the electrical and thermal conductivity, i.e. Wiedeman Franz law.

Ohm's law: This part is not in Kittel but we discuss it typically in our EMT courses. When an electric field is applied to conductor, the electric field will result in a force on the charge carriers, i.e. the free electrons:

$$\vec{F} = q\vec{E}$$
[2]

This force will result in an acceleration, i.e.

$$a = \frac{F}{m}$$
[3]

So the speed of the free electrons will increase with time until they are stopped by a collision with an imperfection in the crystal structure. Imperfections are lattice vibrations (note that there are many of them at high temperature), impurities (we consider two type of impurities, i.e. (1) substitutional impurities this are atoms that replace an atom of the crystal structure and are sitting at the same position in the lattice; (2) interstitial impurities which are atoms that are in between the atoms of the lattice, they normally reside in the larger voids (an interstitial often creates local strain in particularly when it is a little larger than the void it resides in)), vacancies (i.e. missing atoms), and crystal boundaries. Students interested in various types of imperfections are encouraged to take Dr. Piner's MSE7401 course. The effect of these imperfections is that on average a free electron will only accelerate for an average time of τ seconds. This time is often referred to as the mean time between collisions and of course τ is proportional to the mean free path length (l_o) of the free electrons and the speed of the electrons, i.e.

$$\tau = \frac{l_o}{v_{thermal}}$$
[4]

Note that the denominator says the thermal velocity of the electrons. This is not the drift velocity of the electrons because of the applied electric field. It is the speed of the electrons at temperature T. Kinetic theory provides an expression between T and $v_{thermal}$, i.e.

$$v_{thermal} = \sqrt{\frac{8k_B T}{\pi m}}$$

[5]

This speed is much, much higher than the drift velocity caused by the field. At room temperature the thermal speed of the electrons is on the order of 10^5 m/s while the drift velocity due to the applied electric field is on the order of mm/sec or cm/s. So the mean time between two collisions is essentially independent of the drift velocity and only depends on the thermal velocity which is of course proportional to the absolute temperature. The figure below shows both the thermal velocity and drift velocity for a current flowing through a copper wire.
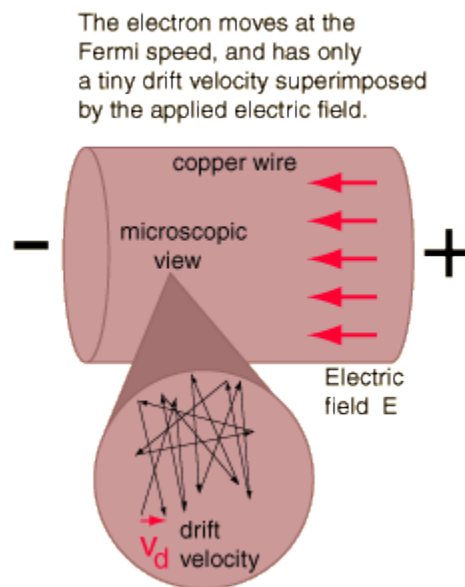

Fig. 1: thermal and drift velocity of free electrons in a copper wire exposed to an electric field E.

We find now for the average drift velocity of the electrons

$$v_{average} = \frac{1}{2}\frac{qE}{m}\tau$$

[6]

Which with equation [4] changes to:

$$v_{average} = \frac{1}{2}\frac{qE}{m}\frac{l_o}{v_{thermal}}$$

[7]

The total current density should be proportional to the concentration of the charge carriers (n), the charge of the charge carriers (e), and the drift velocity of the charge carriers (v_{drif}) thus:

$$J = env_{average} = en\frac{1}{2}\frac{eE}{m}\frac{l_o}{v_{thermal}} = \sigma E \qquad \text{with } \sigma = \frac{n}{2}\frac{e^2}{m}\frac{l_o}{v_{thermal}} \qquad [8]$$

Or in other words the current density is linear proportional to the applied electric field, i.e. Ohm's law. The n is the concentration of free charge carriers in the material which is equal to the concentration of atom (N) times the number of free electrons per atom (f). So σ can be written as:

$$\sigma = \frac{Nfe^2 l_o}{2mv_{thermal}} \qquad [9]$$

Wiedeman-Franz law: Although we saw in chapter 5 that thermal conductivity in materials can be caused by the motion of phonons, for metals mainly the motion of electrons will contribute to the thermal heat conductivity, i.e. an electron that moves from side A of the crystal to side B will carry with it its kinetic energy. The Wiedemann-Franz law states that the ratio of thermal conductivity and electrical conductivity in metals is linear with temperature, i.e.

$$\frac{\kappa}{\sigma} = LT \qquad [10]$$

Note that in chapter 5 we found that the thermal conductivity is proportional to average velocity of the particles and the concentration of the particles (see text below equation (42) on page 122). Equation [8] above told us that the electric conductivity is inversely proportional to the average velocity of the particles and some other stuff that is independent of T. So we can rewrite equation [10] to

$$\frac{\kappa}{\sigma} \sim \frac{C_1 v_{thermal}}{C_2 / v_{thermal}} = \frac{C_1}{C_2} v_{thermal} \sim LT \qquad [11]$$

There are also things that we cannot understand from the free electron model, such as the heat capacity and the magnetic susceptibility of the conduction electrons.