Mock Exam Chapter 4-5, Kittel.

1. Describe in your own words the following terms:
   a. Harmonic approximation
   b. Dispersion relation
   c. Longitudinal modes
   d. Elastic constants

2. What characteristics must the crystal structure of a solid have in order for optic modes to exist? Is it possible for the phonon dispersion relations of a solid to contain acoustic modes only? If so what characteristics are required for the crystal structure.

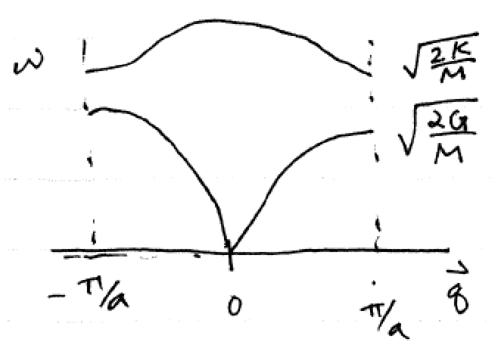
3. Describe in your own words the following terms:

a. Planck's distribution.
b. Phonon density of states (density of modes).
c. Einstein Frequency and Einstein Temperature
d. Heat current
e. Umklapp processes.

4. Discuss the physics underlying the Debye model. What is the primary assumption that this model makes about the behavior of the phonon frequencies as a function of the wave-vector? Briefly contrast this with the assumption made in the Einstein model.

5. Consider 1-d chain with identical masses M. Assume that there are nearest neighbor springs with spring constant K_1 and next-nearest neighbor springs with spring constant $K_2<K_1$.
   a. Find the dispersion relation for this system.
   b. Calculate the speed of sound and compare to a system without next-nearest neighbor interactions.

6. Assume a system consisting of atoms of the same mass but with alternating spring constants, K and G between the atoms. The dispersion relation of such system is given in the figure below.



As you can see above there is a gap in the density of states, i.e. a frequency range for which there are no stationary normal model. How can one change K, G, and M to make the gap larger or smaller.

7. Consider and aluminum sample. The nearest separation 2R (2x atomic radius) between the Al-Al toms in the crystal is 0.286 nm. Taking a to be 2R, and given the sound velocity in Al as 5100 m s^{-1}, calculate the force constant C in Equation (1) of chapter 4. Use the group velocity vg from the actual dispersion relation, i.e. equation 7 of chapter 4, to calculate the sound velocity at a wavelengths of λ=1 mm, 1 um, and 1 nm. What do you conclude?

8. Aluminum has a Debye temperature of 394 K. Calculate its specific heat in the summer at 25 °C and in the winter at -10 °C. Note that the heat capacity below the Debye temperature significantly depends on the T (see also Fig. 4.43 of the handout).

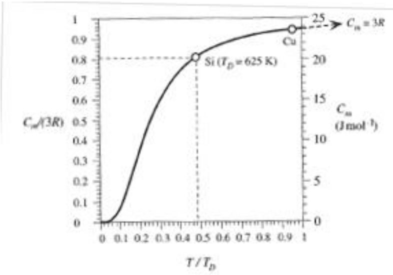


Figure 4.43 Debye constant-volume molar heat capacity curve. The dependence of the molar heat capacity C_m on temperature with respect to the Debye temperature: C_m versus T/T_D. For Si, T_D = 625 K, so at room temperature (300 K), T/T_D = 0.48 and C_m is only 0.81 (3R).

9. A. Given that silicon has a Young's modulus of about 110 GPa and a density of 2.3 g cm^{-3}, calculate the mean free path of phonons in Si at room temperature.
   B. Diamond has the same crystal structure as Si but has a very large thermal conductivity about 1000 W m^{-1} g^{-1}, Young's modulus Y of 830 GPa, and density ρ of 0.35 g cm^{-3}, calculate the mean free path of phonons in diamond.

Table 4.5 Debye temperatures T_D, heat capacities, and thermal conductivities of selected elements

	Crystal							
	Ag	Be	Cu	Diamond	Ge	Hg	Si	W
T_D (K)*	215	1000	315	1860	360	100	625	310
C_m (J K^{-1} mol^{-1})†	25.6	16.46	24.5	6.48	23.38	27.68	19.74	24.45
c_s (J K^{-1} g^{-1})†	0.237	1.825	0.385	0.540	0.322	0.138	0.703	0.133
κ (W m^{-1} K^{-1})†	429	183	385	1000	60	8.65	148	173

*T_D is obtained by fitting the Debye curve to the experimental molar heat capacity data at the point $C_m = \frac{1}{2}$ (3R).
†C_m, c_s, and κ are at 25 °C.
SOURCE: T_D data from J. De Launay, *Solid State Physics*, vol. 2, F. Seitz and D. Turnbull, eds., Academic Press, New York, 1956.

10. Consider a one-dimensional crystal with three atoms in the basis. You may assume the force constant between all atoms is the same.
a) How many phonon branches do you expect. Give a qualitative justification for your answer.
b) Support your answer to part a) mathematically. Note that you do not need to find dispersion relations. You only need to develop your mathematical model to a point where it supports your answer to part a).