HW6_1

1. The kinetic energy of a free electron is given by its kinetic energy, i.e.:

$$\varepsilon_k = \frac{\hbar^2}{2m} k^2$$

Determine the expectation value of the energy of a free electron gas at T=0 with a Fermi-energy ε_F. Use the same approach as we used to determine the expectation value of the x on page 120, i.e.

$$\langle \varepsilon \rangle = \frac{\iiint\limits_{V_k} d\tau_k \ \varepsilon_k D(\vec{k})}{\iiint\limits_{V_k} d\tau_k D(\vec{k})}$$

Where V_k is the volume of k-space within the Fourier-energy (i.e. $|\mathbf{k}|<k_F$) and $d\tau_k$ is a volume element of k-space. Note that this equation says weighted average over k-space of the energy of the different modes divided by number of modes.

A simpler method to obtain the same thing is to use the density of states in terms of energy and to calculate the total energy in the states below ε_F and divide that by the number of particles. So calculate:

$$\frac{U}{N} = \frac{1}{N} \int_0^{\varepsilon_F} \varepsilon D(\varepsilon)\, d\varepsilon$$

2. Determine the density of state of a 2D electron gas. Note that this problem is more or less similar to the problem to determine the density of states of 2D phonon gas, however it differs because the dispersion relation for phonons is different from the dispersion relation for free electrons.
Hint: You might want to follow the 3D problem discussed on pages 138-140 in Kittel.

3. Derive an expression for the Fermi energy for a 2D electron gas in terms of electron concentration. Note that the electron concentration in 2D reads electrons per surface area, i.e. N/A.