2. Gauss' Law [1]

Purpose: Theoretical study of Gauss' law.

Equipment: This is a theoretical lab so your equipment is pencil, paper, and textbook.

When drawing field line pattern around charge distributions we use in general three different rules:

(a) Field lines start at positive charges or in infinity and end at negative charges or in infinity.
(b) Field lines never cross each other. Crossing field line would mean that at the position of the crossing field lines the direction of the electric field is no longer unambiguous determined (note that the electric field direction is tangent to the field lines).
(c) The field line density is linear proportional to the electric field. So the electric field is larger in areas with a lot of field lines.

PROCEDURE:

1. The next few questions involve point charges.

(a) Draw the electric field lines in the vicinity of a positive charge +Q. Pay attention to the rules provided above.

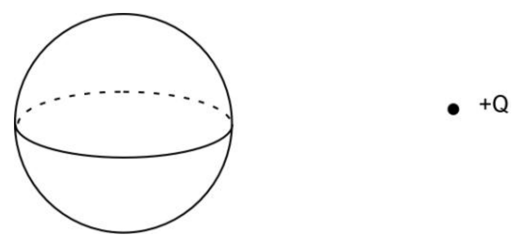
(b) Do the same for a negative charge –Q. Pay attention to the rules provided above.

(c) Do the same for a positive charge of +2Q. As the electric field around the charge will be twice as large as the electric field around a charge of +Q, the field line density will have to be twice as large. Make sure that you use twice as many field lines as for the field line pattern of problem 1(a).

(d) Comment whether or not all field line rules provided above are obeyed.

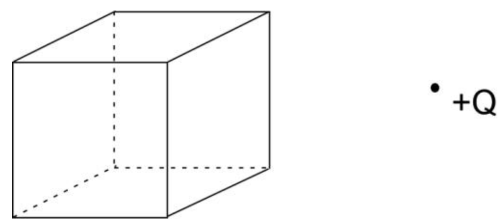
2. Consider a Gaussian surface, i.e. sphere, outside of which a charge $+Q$ lies. Note that this sphere is not an object, just a virtual surface defining a Gaussian volume.

(a) Draw several electric field lines from $+Q$. Now only consider the field lines intersecting with the sphere.



(b) Looking back at your drawing, field lines impinge upon the spherical surface from the outside heading inward (this is defined as negative flux) and eventually impinge upon another part of the surface from the inside heading outward (positive flux). Does it seem reasonable that the total number of field lines that enter the sphere is the same as the total number of field lines that leave the sphere? Explain! If we count the field lines leaving the sphere as <u>positive</u> and the field lines entering the sphere as <u>negative</u>, is the <u>net</u> number of field lines entering or leaving the sphere is zero?

(c) Suppose we replace the virtual sphere with a virtual cube. Would the net total number of field lines leaving the surface of the cube still be zero (again count entering field lines as negative and leaving field lines as positive)? Note we assume that the charge is situated outside the cube. Explain!



The number of field lines can be calculated from the number of the field line density, i.e. the number of field lines per surface area. :

$$\#\,\text{Field lines} = \oiint_S \text{Field Line Density}\, dA$$

Where S is the surface of the sphere or the cube. As the field line density is proportional to the electric field, the number of field lines through the surface can be written as:

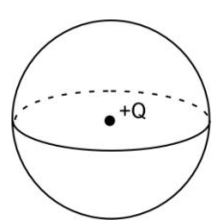
$$\#\,\text{Field lines} = \oiint_S \text{Intersecting Field Line Density}\, dA \sim$$

$$\oiint_S \text{Component of E perpendicular to surface}\, dA \sim \oiint_S \vec{E} \bullet d\vec{A}$$

The last integral is called the Electric flux through the area S. The vector $d\vec{A}$ defines an infinitesimal small part of S. The length of this vector is proportional to the surface area of the infinitesimal small part of S. The direction of this vector indicates the orientation of the infinitesimal small part of S in space (see also Fig. 23-3 in the textbook).

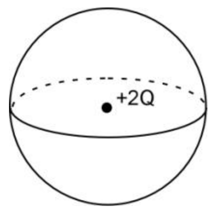
3. Now consider a Gaussian sphere centered on +Q

(a) Draw some electric field lines. Make them long enough to intersect the sphere.



(b) Is the total electric field flux through the sphere, zero, positive, or negative? (leaving field lines contribute to a positive flux and entering field lines to a negative flux) Does this make sense, considering the charge enclosed? Discuss.

(c) Now consider that the charge is doubled to +2Q. Draw the field line pattern.



(d) What happened to the total number of field lines intersecting the sphere?

(e) What happened to the Electric flux going through the sphere?

(f) In symbols, give an expression for the total electric flux (Φ_E) through the sphere? Assume that the radius of the spherical Gaussian surface is r. Notice that in this case the electric field lines intersect the sphere perpendicular to its surface and furthermore that the electric field strength is uniform over the surface.

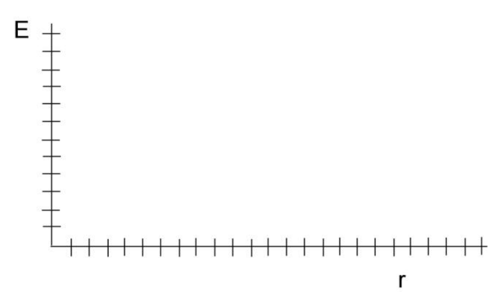
4. We still consider the point charge of question (3). You saw above that the number of field lines (read the total flux) through a closed surface is linear proportional to the electric flux through the surface. We furthermore saw that the number of field lines leaving an enclosed surface is linear proportional to the amount of charge enclosed by that surface. So more charge enclosed means more flux through the surface enclosing the charges. This statement is called Gauss' Law:

- in words:
  The electric flux through a closed surface is equal to the total charge enclosed by the surface divided by ε_o[1].
- in symbols: $\oiint_S \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

(a) How do we know that the electric field strength does not vary over the spherical surface?

(b) Because of the simplifying conditions discussed in part (4a), we can apply Gauss' Law to find the electric field due to the enclosed point charge. Find the electric field at a distance r from a point charge +Q. Use Gauss' law. Start of by assuming the Gaussian surface S is a sphere with radius r around the point charge. Assume that the point charge is centered at the middle of the sphere[1].

(c) Graph |***E***| (the magnitude of E) versus r. [Assume both axes have linear scales.]



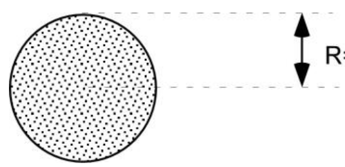
[1] Recall that the surface area of a sphere is $4\pi r^2$

(d) If we had picked a cube as our Gaussian surface instead of a sphere, would it still have been easy to determine the total flux through the surface from Gauss' law? What about using Gauss' law to calculate the electric field strength at any point on the surface of the cube? Explain.

5. The previous problem was mathematically fairly simple. Here's another problem requiring Gauss' Law.

> NOTE: Keep your results in symbolic form and only substitute in numbers when asked for a numerical result. Also, pay careful attention to the distinction between the radius of the sphere, *R*, and the distance, r, from the center of the sphere at which you are evaluating E.

Consider a small sphere (an actual sphere, not a Gaussian surface) of radius $R = 0.1$ m that is charged uniformly throughout its interior. So for $r<0.1$ the charge density is constant and given by ρ=6E-12 Coulomb/m^3. Of course, for r greater than $R=0.1$ *m*, the charge density is zero.



Apply Gauss' Law to find an expression for E when r is less than R. Start off with choosing a Gaussian surface.

(a) First find an expression for the charged enclosed within that Gaussian surface[2].

(b) Second find an expression for the flux going through your enclosed surface. Start off with the integral definition for flux and realize that maybe the electric field is constant across you Gaussian surface and maybe in this case also perpendicular to the surface, so the dot product can be replaced by …..

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[2] Hint: the volume of a sphere is $4/3\pi r^3$

(c) Now use the answers in (a) and (b) to find an expression for E using Gauss' law.

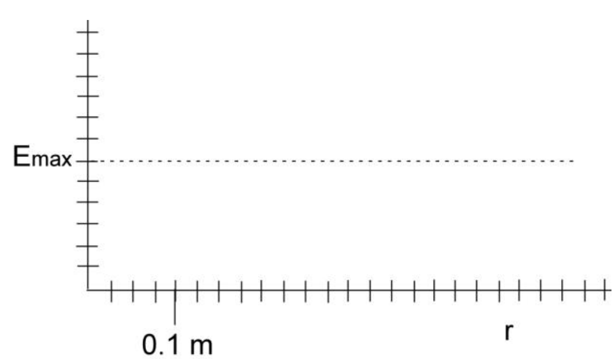
(d) Calculate E at $r = 0.05$ meter? [Your answer should be in N/C.]

(e) Determine the total charge inside the whole charged sphere (i.e. r=R).

(f) Apply Gauss' Law to find an expression for E when r is greater than R. Use the same approach as outlined above, i.e. first determine the enclosed charge, then determine an expression for the flux, and then dunk it using Gauss' law.

(g) What is E at r=5.0 m?

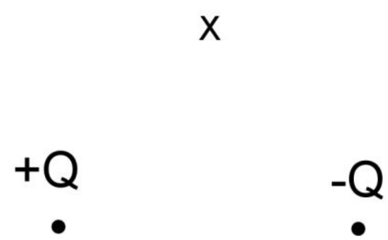
(h) Make an approximate graph of E versus r on the axes shown below



(i) Outside the sphere, how does E compare to E of a point charge of that magnitude? Verify for $r = 5.0$ m. Assume that the point charge has a charge equal to the total charge of the sphere that you calculated in 5(e).

(i) On your graph in part (g) above, use a dotted line to represent the electric field if the sphere shrank to a point charge but still contained the same total charge.

6. Consider two point charges of opposite sign as shown:



(a) In what direction is the total electric field at the point indicated by the X? Draw a vector on the sketch above at the point X to represent E. Also show how E_{Xtot} is the vector sum of the electric field, E_{X+} caused by the charge +Q and the electric field E_{X-} caused by the charge –Q. [Hint: Use vector addition.]

(b) On the sketch below, draw several (at least ten) electric field lines for the configuration above. This should be enough to indicate E in much of the vicinity of the two charges.

(c) On your drawing above, add a spherical Gaussian surface that encloses both charges, centered on the point midway between the charges. Make it fairly large, but be sure several of the electric field lines penetrate the Gaussian sphere. Add more field lines as needed.

(d) Look carefully at your drawing above. In what direction does E point (inward? outward? tangent?) at various locations on the Gaussian surface? Redraw your Gaussian surface below and draw short arrows on the surface indicating the direction of E on the Gaussian surface.

(e) Consider the following argument from a student who is trying to determine E somewhere on the previous Gaussian surface:

> "The total charge enclosed by the surface is zero. According to Gauss' law this means the total electric field flux through the surface is zero. Therefore, the electric field is zero everywhere on the surface."

Which, if any, of the three sentences are correct?

Explain how the student came to an incorrect conclusion.

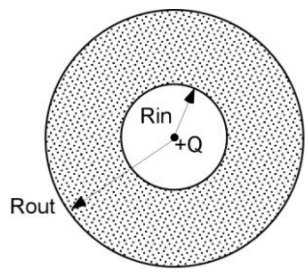
(f) Refer back to 6(a). If you were asked to calculate the electric field at the place of the X, would you attempt to apply Gauss' Law or would you use another method? Discuss.

> Gauss' law is always true, no matter how complicated the distribution of electric charges. In fact it's even true when the charges are in motion. However, it's rare to find a situation with enough symmetry that applying Gauss' law becomes a convenient method to calculate the electric field.

7. Electric fields in the vicinity of a conductor

In class we discussed two types of materials, i.e. insulators and conductors. Conductors contain free electrons that are not bound to the atoms, but can move freely through the material. An example of a conductor is the copper or aluminum wiring used in your house and that connects the switches to the lamps. In insulating materials the electrons cannot move and are bounded to the atoms.

Assume a point charge +Q lies at the center of an uncharged, hollow, conducting spherical shell of inner radius R_{in} and outer radius R_{out} as shown. Your ultimate goal is to find the electric field at all locations for this arrangement.

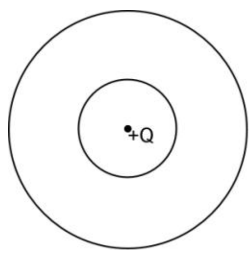


(a) Why is it important to know whether the shell is a conductor or insulator? Is there any real difference?

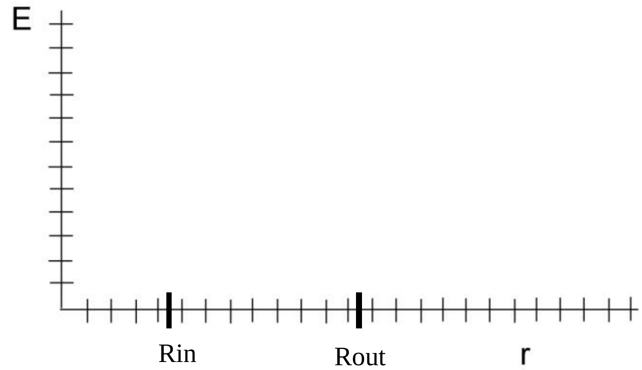
(b) What does the charge distribution within the shell itself (between R_{in} and R_{out}) look like: (I) before the point charge +Q is inserted? (Draw a picture); (II) with the point charge +Q in place? (Draw a picture); (III) Are the charge distributions the same? Why or why not?

(c) What is the electric field within the shell itself (between R_{in} and R_{out} NOT including the surfaces: (I) before the point charge $+Q$ is inserted?; (II) after the point charge $+Q$ is inserted?; (III) How did the fact that the shell is a conductor help you answer the two previous questions?

(d) We are still considering the conducting shell with the point charge in the center. Draw electric field lines in the region $0 < r < R_{in}$ (r is the distance from the point charge) and for the region $r>R_{out}$. What are the field lines between R_{in} and R_{out}? Also show the relative amount of charge that has moved to the inner and outer surfaces of the conductor.



(e) Use Gauss' law to find the electric field as a function of r for all three regions, and then graph it below.



(f) Compare the electric field for r just inside R_{in} and just outside R_{out}. Are the magnitudes equal? Explain.

Reference: [1] This lab was adapted from a lab developed at the Physics Department of the University of Virginia, PHYS2419, Fall 2010.