11. AC-resistances of capacitor and inductors: Reactances.

Purpose: To study the behavior of the AC voltage signals across elements in a simple series connection of a resistor with an inductor and with a capacitor.

Equipment: Pasco RLC circuit board, Model 2001 AC generator, dual trace oscilloscope.

Earlier this semester when we discussed circuits, we always assumed that the batteries, i.e. the voltage sources were providing a constant electric potential across its terminals. If a circuit only consists of resistors such a voltage source creates a constant current through the components of the circuit. We refer to such a circuit as a DC-circuit. All the voltages across its components are referred to as DC voltages and all the currents through its components are referred to as DC currents. DC is the acronym for Direct Current. If we ignore the internal resistance of a battery, a battery can be considered to be a DC voltage source, i.e. it provides a constant electric potential across its terminals. Voltage sources however, are not always DC voltage sources. Some power adapters provide an <u>A</u>lternating <u>V</u>oltage across its terminals. We refer to such a voltage source as an AC voltage source.

The power strips in your dorm-room for example provide an AC voltage of 120 volts. So the voltage across its terminals varies as a function of the time with a frequency of 60 Hz and can be described by the following function:

$$V(t) = V_o \sin(2\pi \ f \ t) = 120 * \sqrt{2} \sin(120\pi \ t) = 170 \sin(377t) \qquad [1]$$

We say that the voltage is sinusoidal.

<u>Questions:</u>

1. The voltage out of a wall socket in your dorm room varies according to equation 1a as a function of the time. The maximum value of equation 1 is called the amplitude of the voltage. What is the amplitude of the voltage?

2. The voltage is sinusoidal, so periodic. The time for the sine function to go from zero to one, back to zero, to minus one, and then back to zero again, is called the period. How large is the period of the sinusoidal voltage described by equation 1 in seconds?

Note that the 120 volt of a standard outlet does not refer to the amplitude of the AC-voltage. The 120 volt is called the rms value of the voltage. The average power dissipation in a resistor connected to an AC power supply that provides an RMS value of 120 volt is the same as the power dissipation in a resistor connected to a DC power supply that provides 120 volts. You will learn more about this in Apllied Electronics, a physics elective, or in some of your electrical Engineering courses.

When an AC voltage is applied across the terminals of a resistor, the current through the resistor will also vary as a function of the time. In Chapter 30 we have seen that the relation between the voltage across a resistor and the current through a resistor is linear and is given by Ohm's law.

Questions:

3. Give an expression for Ohm's law and put it in the box below.

[2]

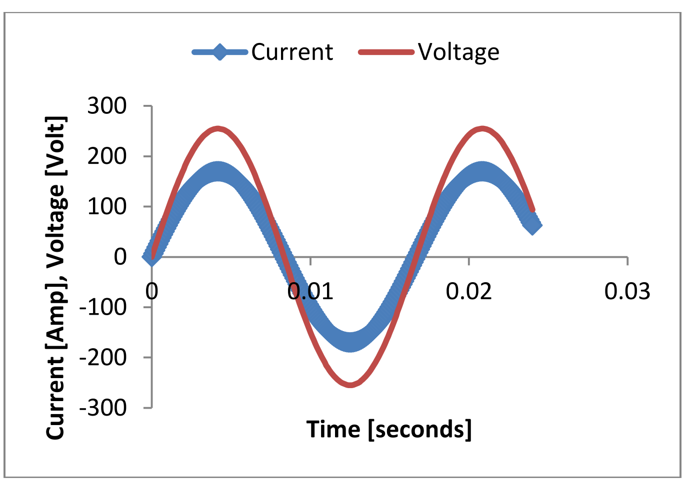
4. Assume that an AC voltage of $V_o\sin(2\pi ft)$ is applied across the terminals of a resistor. Use the answer to question (3) to derive an expression for the AC-current through the resistor.

5. What is the amplitude of the AC current?

6. What is the period of the AC current?

7. What is the frequency of the AC current?

Note that the AC-voltage across a resistor and the AC-current through a resistor are proportional to each other at each moment in time (compare the answer to question (4) with to assume voltage, i.e. $V_o\sin(2\pi ft)$). Both are sine functions. See the figure below.



In chapter 29 we learned about a capacitor. When a dc voltage is applied across the leads of a capacitor, after the capacitor is charged, no currents will flow through the capacitor leads.

Questions:

8. Make a drawing of a capacitor and identify the different parts. Indicate which parts are conductive and which parts are non-conductive.

9. Use your sketch of question 8 to explain why a DC voltage applied across the terminals of the capacitor will not result in a DC-current flowing through the capacitor leads.

On the contrary, when an AC voltage is applied across the terminals of a capacitor, an AC-current flows through the leads of a capacitor. This current transports charge to and away from the capacitor plates and continuously charges and discharges the capacitor. In the following part you will derive an equation for the sinusoidal current in the capacitor leads, assuming a sinusoidal voltage is applied across the terminals of a capacitor.

In chapter 29 we learned the equation that describes the capacitor, i.e. (equation 29.18):

$$\boxed{Q = CV} \qquad [3]$$

So the voltage across a capacitor is linear proportional to the charge on the plates of the capacitor. The more charge is stored on the plates of the capacitor, the higher the electric potential across its terminals. We saw in the RC-lab how we can charge and discharge a capacitor through a resistor.

Questions:

10. In above equation what does Q stands for and what are its units?

11. In above equation what does C stands for and what are its units?

12. Take the derivative of equation 3 towards the time. So take the d/dt from the left side of equation 2 and the d/dt from the right side of equation 3. Note that the capacitance C is independent of the time, so it can be considered to be a constant and factors out of the derivative. What are the units of the left side of the equation you derived? So you can

replace this derivative with what quantity? Place the equation you derived in the box below.

[4]

The equation you derived in question 12 is called the i-v relation of a capacitor. This is no longer a linear equation, but a differential equation as it contains quantities as well as derivatives of quantities.

<u>Questions:</u>

13. Assume that a sinusoidal voltage described by $V_o \sin(2\pi ft)$ is applied across the terminals of a capacitor. Use equation 4 to derive an expression for the AC current going through the capacitor's leads.

14. What is the amplitude of the AC current?

15. What is the period of the AC current?

16. What is the frequency of the AC current?

The ratio of the voltage amplitude to the current amplitude can be considered to be the AC resistance of the capacitor.

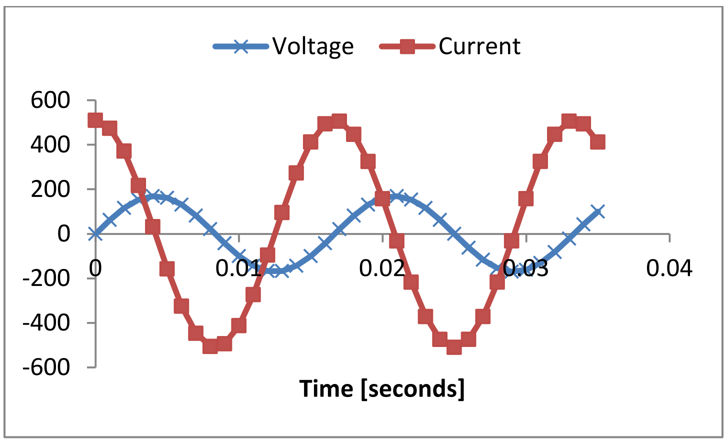
<u>Questions:</u>

17. Derive an expression for the AC resistance of the capacitor by dividing the voltage amplitude by the current amplitude (use the answer on question 14). We often refer to this AC resistance as the reactance of the capacitor and identify it with X. Write the expression for X you derived in the box below.

[5]

18. Note that the AC resistance of a capacitor depends on the frequency. Does the AC resistance of a capacitor increase or decrease as a function of the frequency? Compare the AC resistances of a capacitor with the AC resistance of a resistor (i.e. equation 1b). Is the AC resistance of a resistor frequency dependent?

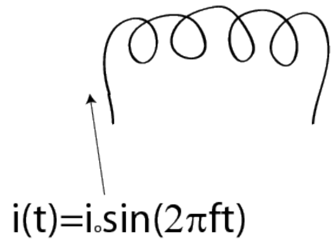
Note that if the AC-voltage across capacitor is a sine function and the current through the leads is a cosine function. So current and voltage are not directly proportional to each other for a capacitor. Since a cosine function can be considered to be a shifted sine function, we speak of a phase shift between current and voltage (see Figure below).



Questions:

19. How much is the phase shift between the voltage across a capacitor and the current through the leads of the capacitor?

Also an inductor has an AC resistance. Note that an inductor can be as simple as a solenoid, i.e. a copper wire wrapped around a toothpick. If the copper wire is thick enough, the solenoid will not have a significant resistance. So putting a large current through the solenoid will not result in a significant voltage drop across the device since its resistance is negligible. However interesting effects happen if the current is not DC but AC. We start of by reviewing the magnetic field inside a solenoid, i.e. chapter 32.



<u>Questions:</u>

20. A current through a solenoid generates a ……..………......……. (Ampere's law chapter 32 pp. 936-938)

21. A <u>changing</u> current through a solenoid that has a cross sectional area A generates a <u>changing</u> (chapter 33.3) ………………..………

22. A changing ……………………….. (substitute here the answer to question 21) causes an induced ………..…….. across the terminals of a solenoid (Faraday's law chapter 3.5).

So in other words a changing current (read AC current) through an inductor will cause an AC voltage across the terminals of an inductor. We called this the induction voltage and we saw an important application of that in last week's transformer lab. To derive an expression for the AC resistance of an inductor we have to start with equation 33-31, i.e. the inductor equation:

$$Li = \Phi_B \qquad [6]$$

This equation tells us that the flux through an inductor is linear proportional to the current through the inductor. It is easiest to consider an inductor to be a single loop of wire.

<u>Questions:</u>

23. In above equation what does Φ_B stands for? What are its units?

24. What is the name of the quantity L? What are its units?

25. Take the derivative of equation 6 on both sides. Note that L is a constant and will factor out. Note that according to Faraday's law $d\Phi_B/dt$, is the induced emf along one winding so $Nd\Phi_B/dt$ is the total induced emf along the copper wire of the inductor and can be replaced with the electric potential difference across the inductor. Write the derived i-v relation of the inductor in the box below.

[7]

Note that also this equation is a differential equation as both quantities as well as derivatives of quantities are included in the equation. Equation [7] might contain a negative sign because of lenz's law. The way we define the current direction in circuits though would cancel out this negative sign. You will learn more about this in applied electronics.

Questions:

26. Assume that a sinusoidal current flows through the inductor. Assume the current is described by the following expression: $i(t)=I_o \sin(2\pi ft)$. Use equation 7 to derive an expression for the AC voltage across the inductor.

27. What is the amplitude of the induced voltage across the inductor?

28. What is the period of the induced voltage across the inductor?

29. What is the frequency of the induced voltage?

30. Use the answers to question 26 and 27 to derive an expression for the AC resistance of an inductor. We often refer to this ac-resistance as the reactance of the inductor and use symbol X. Write the derived expression for the reactance of an inductor in the box below.

[8]

31. Compare the reactance of a inductor with the reactance of a capacitor. Both are frequency dependent. Which reactance will increase as a function of the frequency?

Lab Procedure:

1. Study the Pasco RLC board. This board contains several components including several resistors and a large inductor. Using the wires provided build the LR circuit shown in Figure 1. You might need to use jumper wires. Use the 100 ohm resistor and the large coil on the Pasco RLC circuit board. Be sure to place the iron core into the center of the coil. Use two voltage probes, one connected to input one and the other to input two of the oscilloscope, to measure the voltage across the resistor and the voltage across the inductor. Connect probe one across the 100 ohm resistor and probe two other across the inductor. <u>Make sure that the black ground wire of both signal probes are connected to the same network node, i.e. the node that connects the resistor and the inductor together.</u> So the left side of the resistor is connected to the red alligator clip of probe 1, the right side of the resistor is connected to the black alligator clip of probe 1, the left side of the inductor is connected to the black alligator clip of probe 2, and the right side of the inductor is connected to the red alligator clip of probe 2. The two black wires need to be connected to the same network node as they are connected to each other in the oscilloscope. Not doing so would create a short in your circuit. Let the TA check your circuit. Note that channel 1 displays the negative voltage across the resistor for our circuit. Since the voltage across a resistor is linear proportional to the current through a resistor, channel 1 is proportional to the current through the circuit and thus the current through the inductor. Channel 2 displays the voltage across the inductor. Turn on the signal generator and adjust the frequency to approximately 500 Hz. Adjust the range of the vertical and horizontal axes until well-defined sinusoidal curves are observed for each voltage signal. Do this by adjusting the time base and the vertical sensitivity of each channel.

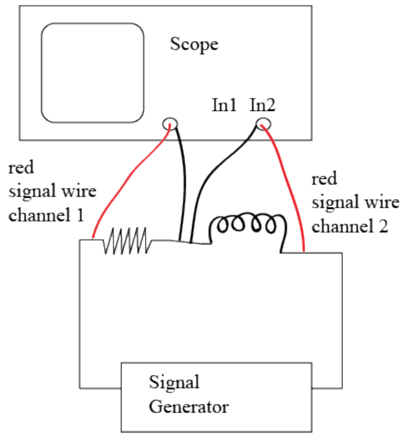


Fig. 1: RL setup (a) schematic diagram (left); (b) setup

Vary the frequency of the generator and see what happens to the voltage across the inductor. You can assume that as you change the frequency, that the current through the circuit stays constant. Since the AC resistance of the coil changes with frequency, the voltage across the inductor will change as you vary the frequency.

Questions:

32. Sketch both signals. Probe 1 shows the time dependence of the negative value of the current, and probe 2 shows the time dependence of the voltage across the inductor.

33. Determine the phase difference (read horizontal shift) between both signals: first in millisecond and then in degrees. Is this in agreement with the answer you gave to question 26? Explain!

34. Verify the frequency dependence of the reactance, i.e. equation 8, by measuring the ac-resistance for two different frequencies. You could do measurements at 500 Hz and for example 2000 Hz. Use the time-base of the oscilloscope to determine the exact frequency. Record the amplitude of both channels.

35. Calculate L from the measurement results of question 34 and equation 8. Note that the amplitude of the current can be calculated from the voltage amplitude of channel 1 and the value of the resistor.

36. Measure the value of L of the coil with the LCR meter. Compare the value with the value obtained from question 35.

37. Remove the iron core from the inductor. Max the amplitude of the function generator. Set the oscilloscope to DC coupling for both channels, trigger on channel 2, make sure the triggering option is auto. Now switch the function generator to output a triangle wave of 500 Hz. Sketch what you see on the oscilloscope. Channel one, i.e. the negative of the current through the circuit is a triangular wave. What wave form best describes the time dependence of the voltage across the inductor? Explain your result using equation 6 and calculus.

38. Now switch the function generator to a square wave of 500 Hz. Sketch what you see on both channels on the oscilloscope. Channel one, i.e. the negative of the current through the inductor shows a square wave. Explain the time dependence of the voltage across the inductor. Explain using equation 6 and calculus.

Note 1: that the RL circuit acts like a circuit whose voltage across the inductor is the derivative of the current. If you consider the current to be the input of the circuit and the voltage across the inductor to be the output, this circuit can be considered a differentiator.

Note 2: the sinusoidal voltage across an inductor and the sinusoidal current through the inductor are not in phase. So you noticed that when the current is maximum the voltage across the inductor is zero and vice versa. We say that current and voltage are 90 degrees out of phase with respect to each other.

Next week's lab:
Next week we meet for the take-out test and the evaluations. Both are mandatory. Furthermore those of you that have missed a lab can make that up next week.

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