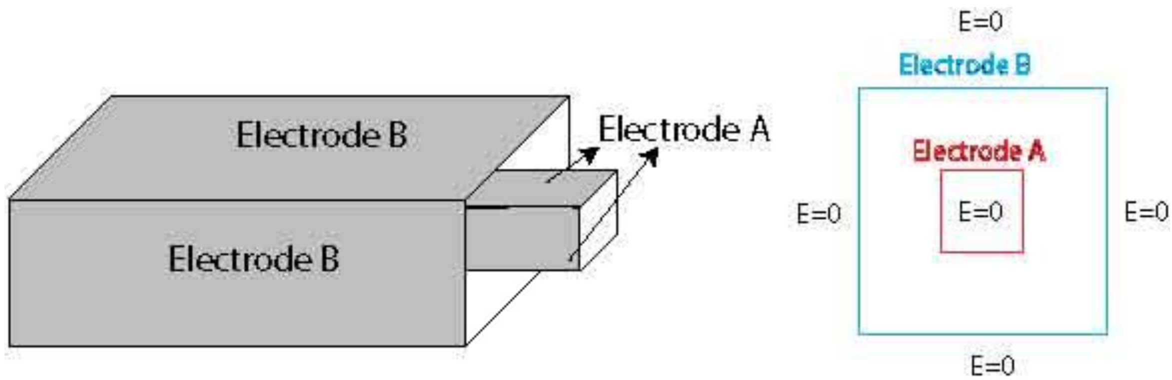
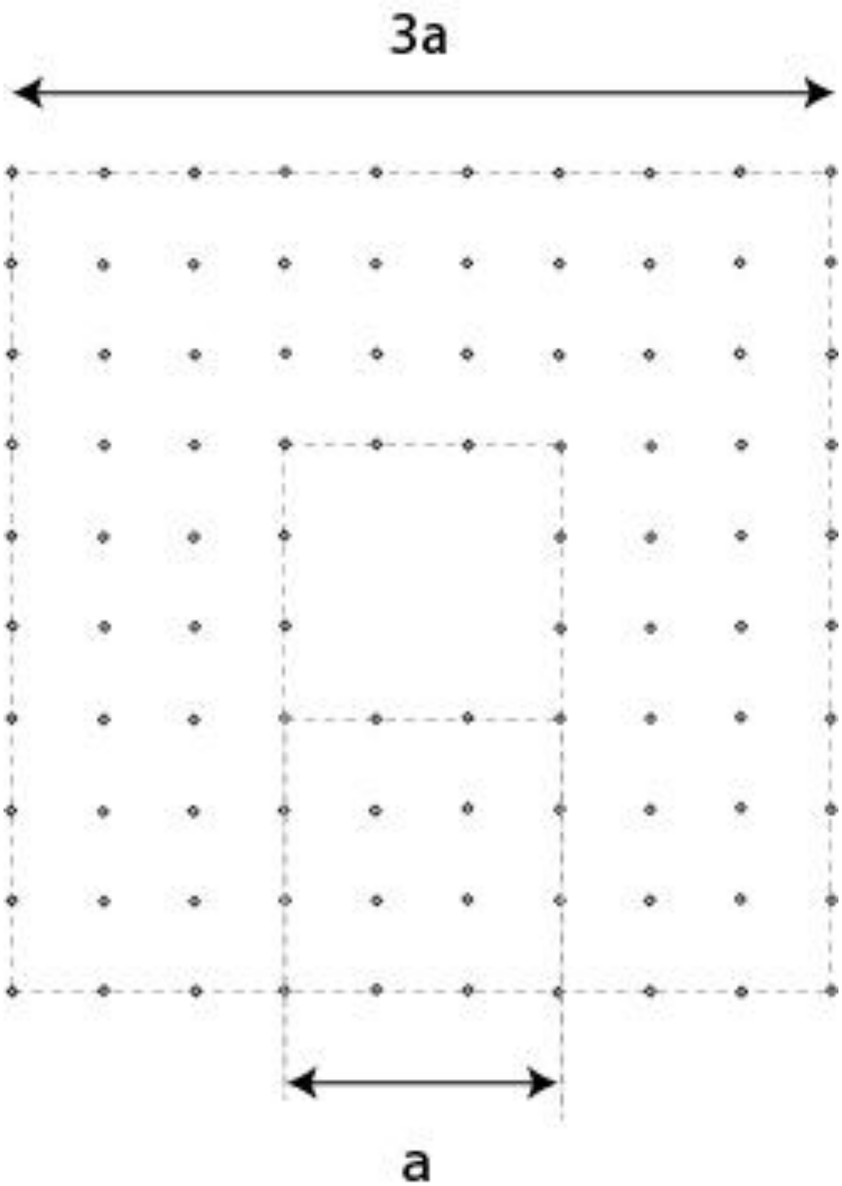
Homework week 6.

1. In this problem we explore how to estimate the solution of Laplace's equation at certain points in space. The points in space are identified by a regular grid.
   Assume a capacitor consisting of an inner electrode (square bar with side a) that is at a potential of 100 Volt and an outer electrode (square bar with side 3a) that is at an electric potential of 0 volt. The space in between the electrodes is filled up with air or vacuum. There is no charge in between the electrodes. The figure below show the two electrodes in grey on the left in a 3D image. The space between the two electrodes is vacuum and thus contains no charge. We assume that the capacitor is infinite long. The figure on the right shows a cross section of the capacitor.



   Note that the electric field enclosed in the area within Electrode A is zero and the electric field outside the outer electrode is also zero. So all points within the red square are at the same potential (VA) and all points outside the blue square are at the same electric potential (VB).

   Figure 1 below shows another cross sectional area of the structure. I omitted the electrodes but added a grid for the space in between the electrodes. Assume that the structure extends to + and – infinity in the direction perpendicular to the paper. The grid indicated in the figure below identifies a regular array of field points, i.e. the locations at which we want to estimate the electric potential.



a. Simplify Poisson's equation for the space between the electrodes.

b. Do you expect the electric potential to vary in the z-direction?

c. We know the boundary conditions. The electric potential is 100 volt at the field points located on the inner square, and 0 volt at the field points located at the outer square. Write those boundary conditions in the figure above, so label each field point on the inner square with 100 and each field point on the outer square with zero. Use a pencil for this and write the potential in small font above each field point. Assume that the initial conditions of grid point in between the conductors is zero.

d. In class we learned that the solution of Laplace's equation at a specific field point (x,y) is the average of the electric potential of the field points around (x,y). The book writes this property with an integral. For the two dimensional case this property can be written as:

$$V(x,y) = \frac{1}{2\pi\, R} \oint_{circle} V dl \qquad [1]$$

Where the line integral is along a circle with radius r centered around (x,y). The factor in front of the integral can be understood if one evaluates $\oint_R V dl$ (evaluated over a circle with radius R) for the situation that V(x,y) is a constant independent of the position. Show that for such case equation 1 is valid.

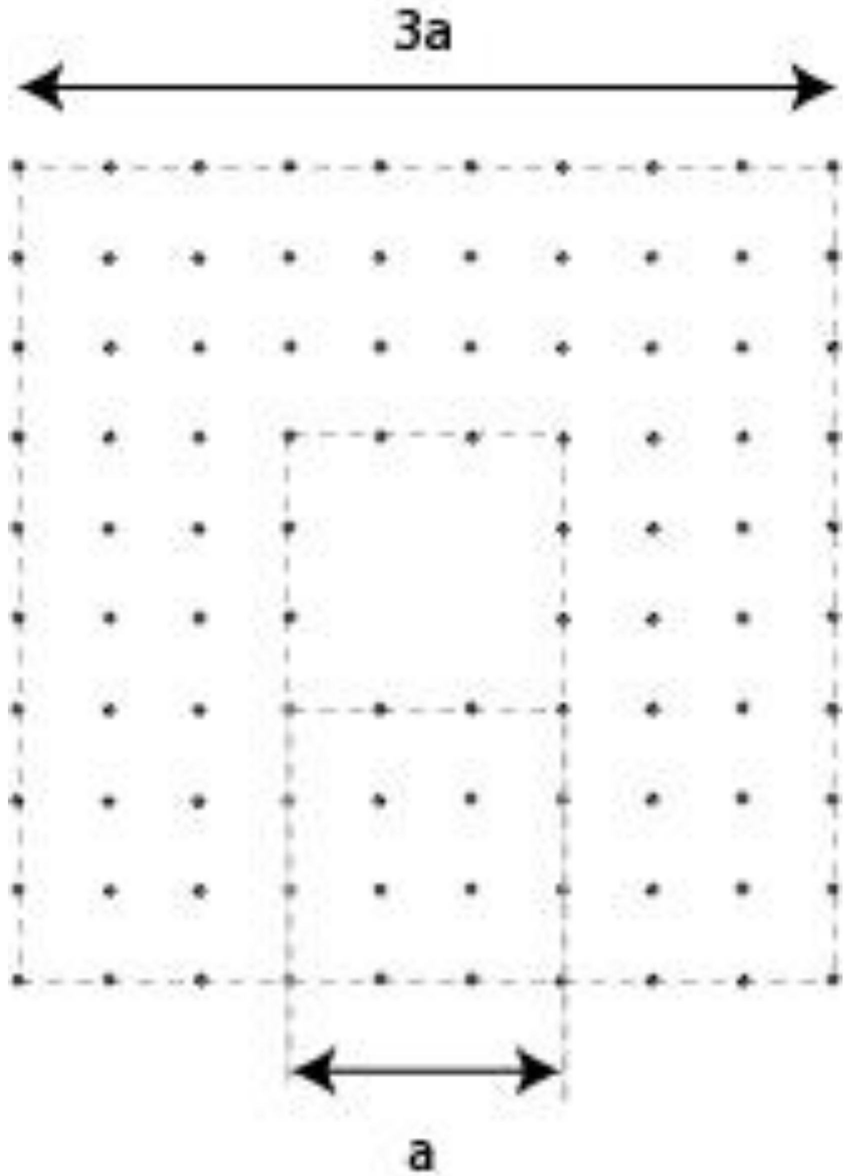
e. As we are only interested in the electric potential at the specified grid points it is possible to rewrite equation (1) using a summation rather than an integral. Write down an expression for the estimate of the electric potential at grid point (k,l) and how this depends on the electric potential at the grid points it is surrounded by (consider only direct neighbor grid points). Use the following notation: V(k,l) is the electric potential at grid point (k,l).

f. Now apply the equation you found for (d) on all grid points in the figure above. Start at the inner conductor using the initial conditions you listed in pencil in the figure above. So for each grid point add the potential of the electric potential of its neighbors and divide by four (round your number down). Write this new electric potential in pen under each grid point (you might want to use a red pen so it is clear which are the initial guess values and which are the new estimates). Remember physicists love symmetry.

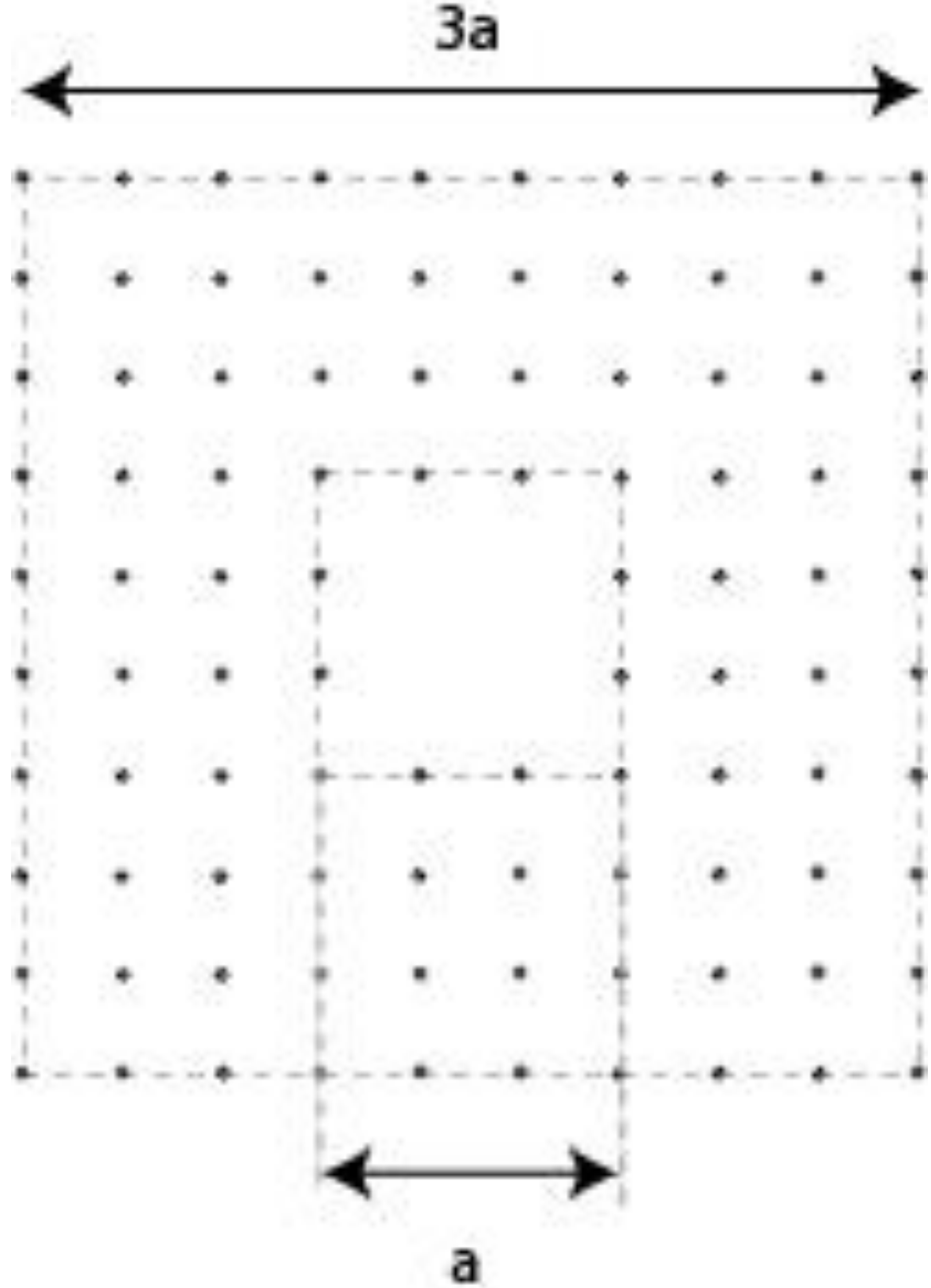
g. Now copy the pen values to the figure below using a pencil and write the values above the corresponding field points. Notice that most of your grid is still zero.

h. Now apply the equation you found for (d) on all grid points in the figure below. Start at the inner conductor using the pencil values you listed in the figure below. So for each grid point add the potential of the electric potential of its neighbor and divide by four (round your number down).

Write this new electric potential in pen under each grid point. Remember <u>physicists love symmetry!</u>



i. Now repeat this process four more times using the sheets provided below. You will see that for each next step, the change of the electric potential values of the field points is smaller. Your last guess will be very close to a good estimate of the electric potential distribution in the space between both conductors.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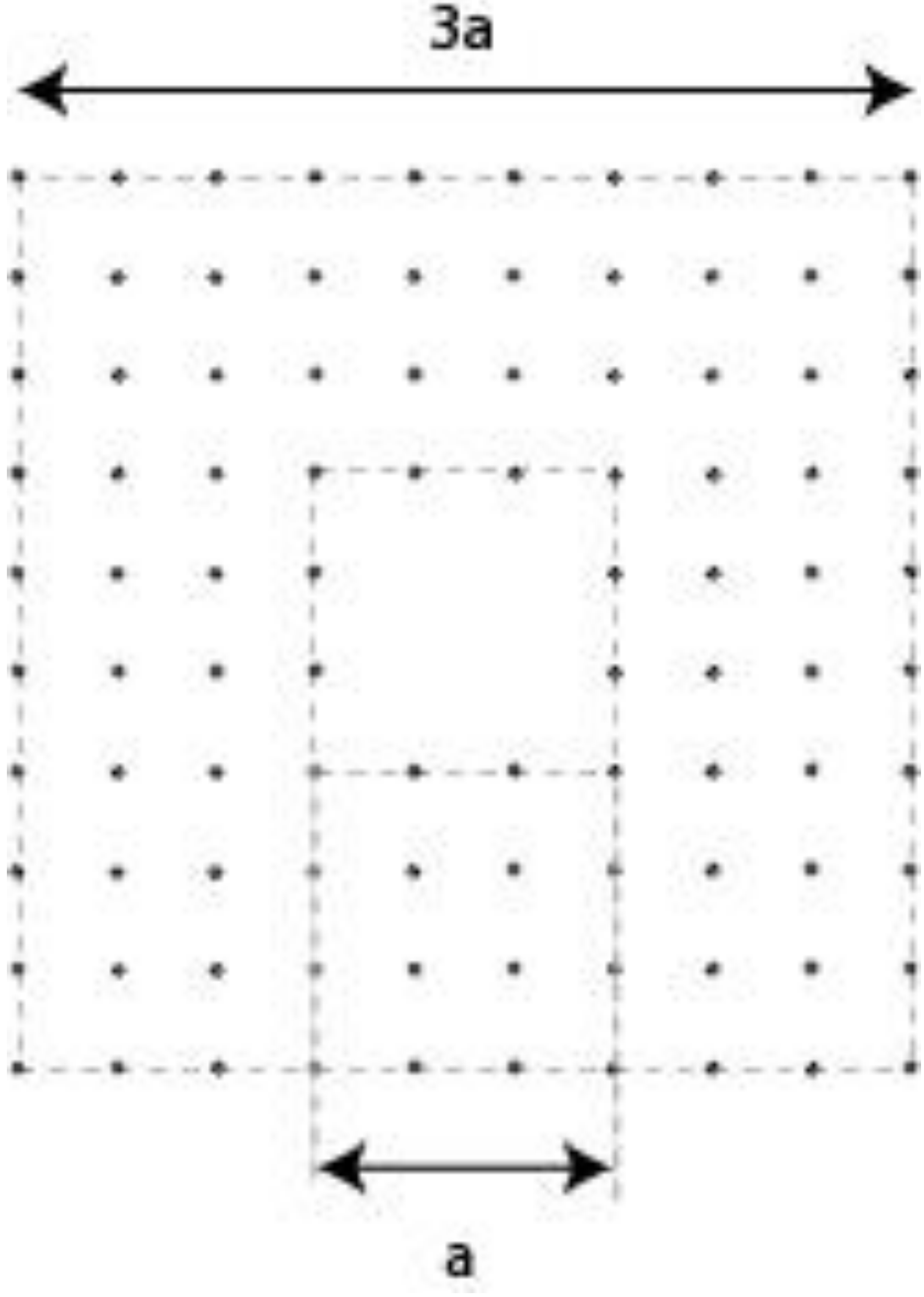
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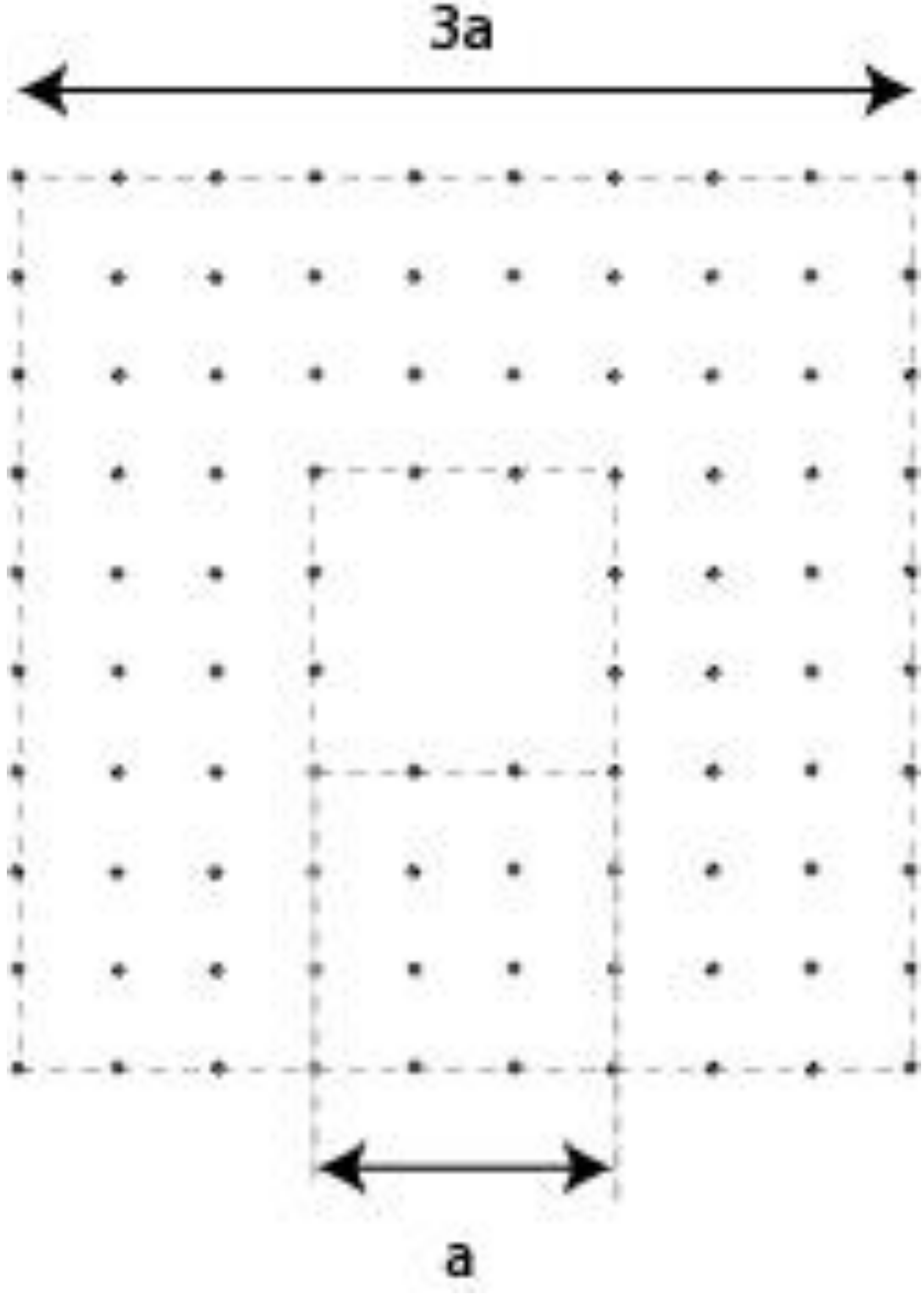
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