Homework Week 4: Gauss law and electric potential electric field relation.

Question 1. E field of infinite plane

1. We know that the electric field everywhere in space due to an infinite plane of charge with charge density located in the xy-plane at $z = 0$ is

$$\vec{E}(z) = \begin{cases} +\dfrac{\sigma}{2\epsilon_0}\hat{z} & z > 0 \\ -\dfrac{\sigma}{2\epsilon_0}\hat{z} & z < 0 \end{cases}$$

(Mentally check that this is true for both positive and negative values of σ.)

(a) Sketch the z-component of the electric field as a function of z.

(b) Draw a similar picture, and write a function that expresses the electric field everywhere in space, for an infinite **conducting** slab in the xy-plane, of thickness d in the z-direction, that has a charge density $+|\sigma|$ on each surface.

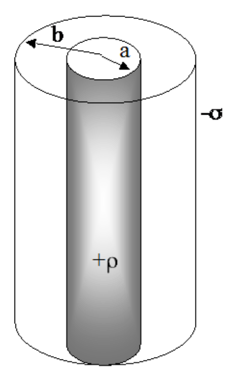
(c) Repeat for a charge density $-|\sigma|$ on each surface.

(d) Now imagine two **conductors**, one each of the two types described above, separated by a distance L. Use the principle of superposition to find the electric field everywhere. Discuss whether your answer is reasonable. Does it agree with the known fact that the electric field inside a conductor is zero? Has superposition been correctly applied? Is Gauss' Law correct? Try to resolve any inconsistencies.

Question 2. Integrate charge density to find Q

The surface charge density on a sphere of radius R is $\sigma = \sigma_0 sin^2(\theta)cos^2(\emptyset)$.
Find $Q = \int dA\sigma(\vec{r})$.

Question 3. Electric field of coaxial cable



A long coaxial cable carries a uniform volume charge density ρ throughout an inner cylinder (radius *a*) and a uniform surface charge density σ on the outer cylinder (radius *b*). The cable is overall electrically neutral. Find **E** everywhere in space, and sketch it.

Question 4. Delta functions and Gauss

The electric field outside an infinite line that runs along the z-axis is equal to $\vec{\mathbf{E}} = \dfrac{2\lambda}{4\pi\varepsilon_0}\dfrac{\hat{\mathbf{s}}}{s}$ in cylindrical coordinates. (This is derived in Griffiths Example 2.1)

a) Find the divergence of the E field for s>0.

b) Calculate the electric flux out of an imaginary "Gaussian" cylinder of length "L", and radius "a" , centered around the z axis. Do this 2 different ways to check yourself: by direct integration, and using Gauss' law)

Question 5. Potential in a uniformly charge solid sphere.
Find the potential inside and outside a uniformly charge solid sphere whose radius is R and whose total charge is q. Use infinity as your reference point. Compute the gradient of V in each region, and check that it yields the correct field (problem 2.21 in Griffiths).

Question 6. Impossible electrostatic fields?
Work problem 2.20 in Griffiths.