Separation of variables assignment 1: Read the first section on Separation of Variables and then work the following two problems.

1. Consider the space described by $0<y<1$ and $x>0$. Assume that the charge density in this space is zero and that the potential at the boundaries of the space is given by the following boundary conditions:
2. $\mathrm{V}=0$ at $\mathrm{y}=\mathrm{o}$
3. $\mathrm{V}=0$ at $\mathrm{y}=\mathrm{a}$
4. $\mathrm{V}=\mathrm{V}_{0} \sin (2 \pi \mathrm{y} / \mathrm{a})$
5. $\mathrm{V}=0$ for $\mathrm{x} \rightarrow$ infinity

Find an expression for $V(x, y)$ by solving Laplace's equation. Work the full problem, so decide how $X(x)$ and $Y(y)$ look like and then apply boundary conditions 1 and 2 to determine constants in your expression for $Y(y)$ and then use boundary condition 4 to determine one of the constants in the expression for $X(x)$. Now combine the $X(x)$ and $Y(y)$ expressions to form a general expression for $\mathrm{V}(\mathrm{x}, \mathrm{y})$ and use boundary condition 3 and the Fourier trick to determine the last constant.

2. Consider the space defined by $0<y<a$ and $-b<x<b$. Furthermore assume the following values for the electric potential at the boundaries:

1. $\mathrm{V}=0$ at $\mathrm{y}=0$
2. $\mathrm{V}=0$ at $\mathrm{y}=\mathrm{a}$
3. $V=V$ at $x=-b$
4. $V=-V_{0}$ at $x=b$

Find an expression for $\mathrm{V}(\mathrm{x}, \mathrm{y})$ by solving Laplace's equation for space given above.


