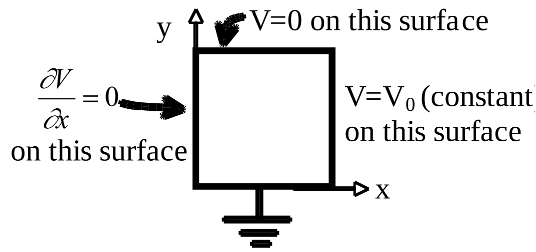
Question 1. Separation of variables- Cartesian 2D

A square rectangular pipe (sides of length a) runs parallel to the z-axis (from -∞ to +∞)
The 4 sides are maintained with boundary conditions given in the figure. (Each of the 4 sides is insulated from the others at the corners)
Note that the left side of the square is the side for which x=0 and the bottom side of the square is defined by y=0. All sides are insulated from each other.



i) Find the potential V(x,y,z) at all points in this pipe. Note that the boundary condition of the left plane requires you to take the derivative of the electric potential, i.e.

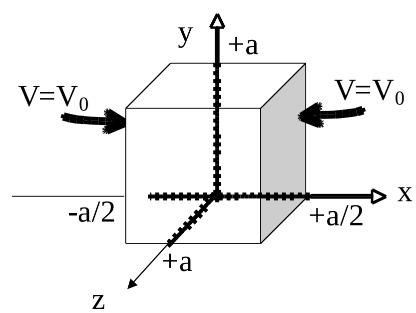
$$\left.\frac{\partial V(x,y)}{\partial x}\right|_{x=0} = Y(y)\left.\frac{\partial X(x)}{\partial x}\right|_{x=0}$$

ii) Sketch the E-field lines and equipotential contours inside the pipe. (Also, state in words what the boundary condition on the left wall means - what does it tell you? Is the left wall a conductor?)

iii) Find the charge density σ(x,y=0,z) everywhere on the bottom conducting wall (y=0). Use one of the boundary conditions of chapter 2, i.e. pp 89 and 90. Notice that E in the bottom conducting wall should be zero.

Question 2. Separation of variables- Cartesian 2D

To be able to work this problem you might first need to study example 3.5 on pages 138-140.
You have a cubical box (sides all of length a) made of 6 metal plates which are insulated from each other.
The left wall is located at x=-a/2,the right wall is at x=+a/2.
Both left and right walls are held at constant potential V=V_0.
All four other walls are grounded.
(Note that I've set up the geometry so the cube runs from y=0 to y=a, and from z=0 to z=a, but from x=-a/2 to x=+a/2 This should actually make the math work out a little easier!)



Find the potential V(x,y,z) everywhere inside the box.
(Also, is V=0 at the center of this cube? Is E=0 there? Why, or why not?)