# Homework Week 9

## Question 1. Potential of non-uniformly charged sphere

a. Rework example 3.8, for a neutral metal sphere in a homogeneous electric field. Find the electric potential outside the sphere. Assume that the metal sphere is held at a potential of V=0. Do not make the same mistake as we did in class, so first use the boundary condition at r=R to derive a relation between Al and Bl. Use the Fourier trick and the boundary condition in infinity to determine the constants.

b. Determine the surface charge density on the surface of the metal.

## Question 2. Potential of non-uniformly charged sphere

a. Suppose that on the surface of a sphere of radius R there is a surface charge density $\sigma(\theta) = \sigma 0 \cos 2\ \theta$, where $\sigma 0$ is a constant. What is V (r, θ) inside and outside the sphere? What is the electric field E(r, θ) inside and outside the sphere? Confirm that the field is discontinuous at the surface of the sphere in accord with the given charge distribution.

b. Now repeat the above exercise for the surface charge density $\sigma(\theta) = \sigma 0 \sin 2\ \theta$. Note that in this case:

$$V(r,\theta) = \begin{cases} \frac{2}{3}\frac{R\sigma_0}{\epsilon_0} - \frac{2R\sigma_0}{15\epsilon_0}\left(\frac{r}{R}\right)^2 P_2(\cos\theta) & r < R \\ \frac{2}{3}\frac{R\sigma_0}{\epsilon_0} - \frac{2R\sigma_0}{15\epsilon_0}\left(\frac{R}{r}\right)^3 P_2(\cos\theta) & r \geq R \end{cases}$$

## Question 3. Spherical

The potential on the surface of a sphere (radius R) is given by $V=V_0 \cos(2\theta)$.
(Assume $V(r=\infty)=0$, as usual. Also, assume there is no charge inside or outside, it's ALL on the surface!)

i) Find the potential inside and outside this sphere.
(Hint: Can you express $\cos(2\theta)$ as a simple linear combination of some Legendre polynomials? )

ii) Find the charge density $\sigma(\theta)$ on the sphere.

## Question 4. work problem 3.17 from the text