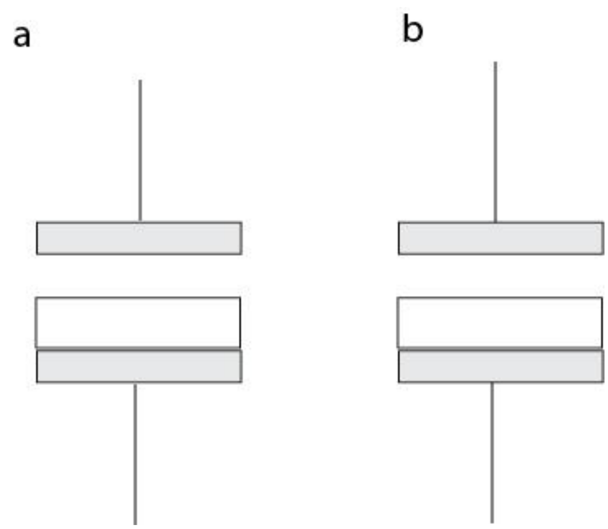
Mock Final Exam
PHYS4310

Those of you who would like to have extra homework credit can work the mock exam and submit it at the day of the final. I will count each correct problem as 1 point towards your homework grade.

1. Suppose you have a parallel plate capacitor that is half filled with dielectric material of dielectric constant ε_r. The area between the dielectric material and the upper plate is filled with air. Assume the parallel plate capacitor has plates with a surface area A that are separated by a distance d. Assume that the free charge density on the top plate is σ C/m^2 and the free charge density on the bottom is $-\sigma$ C/m^2.

    a. Find the electric displacement **D** between the dielectric and the upper plate. Show your work.
    b. What is the electric displacement **D** in the dielectric? Explain.
    c. Derive expressions for the electric field in the dielectric and in the air gap. Show your work.
    d. Determine the polarization **P** in the dielectric and in the air gap. Show your work.
    e. Find the electric potential V between the plates. Show your work.
    f. Identify the location and sign of the bound charges in the figure a below.
    g. Draw the field lines (density and direction) in figure b below. Ignore the fringe fields. Make sure that your drawing is consistent with your answer on question (c).



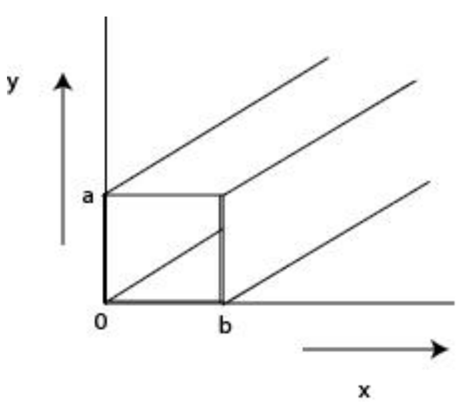
2. Consider a space with an electric field given by the following equation:
$\vec{E} = x^3\hat{i} + y\hat{j} + 0\hat{k}$

   a. Calculate the curl of this electric field.
   b. Is it possible to write this vector field as the gradient of a scalar field? In other words, does an electric potential ϕ exist so that $\vec{E} = -\vec{\nabla}\varphi$? Explain why or why not.
   c. If φ exists calculate φ. Assume $\varphi(0,0,0)=0$.
   d. Calculate the charge distribution ρ that is responsible for above given electric field.

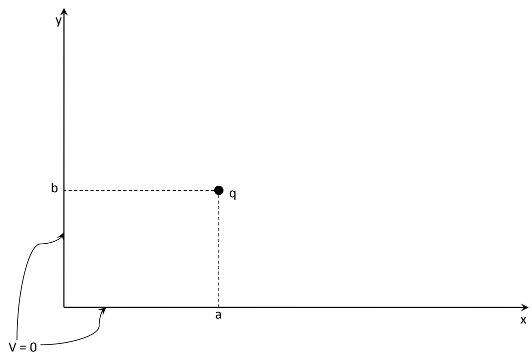
3. Consider an insulating plane through the origin stretched out to infinite in the x-direction and stretched out to b and –b in the y-direction. Assume the plane has a homogeneous surface charge σ C/m^2. Assume a field point located on the z-axis at distance z above the plane.
   a. Find an expression for the contribution of the electric field at point P caused by an arbitrary source point of surface area dx'dy' at position (x',y'). Make a drawing that includes all of your definitions and show your work.
   b. Do the integration and calculate the electric field at point P.

4. Consider the following four infinitely long grounded metal plates as depicted in the figure below. The left, the bottom, and the right plate are all grounded and set at zero volt. The top plate is kept at an electric potential of V_o volt. Use separation of variable to find the solution of Laplace's equation within the waveguide.

   a. What form does the solution of Laplace's equation in the x-direction have? Explain! Use the boundary conditions in the x-direction to simplify the equation. Show your work.
   b. What form does the solution of Laplace's equation in the y-direction have? Explain! Use the boundary condition at y=0 in to simplify the equation. Show your work.
   c. Why can X(x) and Y(y) not be both trigonometric? Explain for math point of view or by using the "extreme-theorem" discussed in the beginning of chapter 3.
   d. Use the Fourier trick and the fourth boundary condition at y=a to solve for V(x,y) within the structure.



5. Two semi-infinite grounded conducting planes meet at right angles. In the region between them, there is a point charge q situated as shown below.
   A. Set up the image configuration and calculate the potential in this region. Indicate which image charges you need and where they should be located and explain in terms of boundary conditions?
   B. What is the force on q? Magnitude and direction.
   C. What is the charge density at point (a,0)?



6. A sphere with a radius R carries a polarization

   $$\vec{P}(\vec{r}) = k\vec{r}$$

   Where k is a constant and **r** is the vector from the center. Determine the bound surface charge σ_b and ρ_b.

7. Find the total electrostatic energy stored in a uniformly charged sphere of radius R and total charge Q. (Note: this is uniform throughout the whole volume; it's not a shell.)

8. A cylindrical conductor of radius a and length l carries a charge q. It is surrounded by linear dielectric material of susceptibility χ_e, out to radius b. A second cylindrical conductor is wrapped around the dielectric material. The two conductors separated by the dielectric material forms a cylindrical capacitor. Assume that the charge on the capacitor plates is Q and –Q.
   c. Determine the total energy stored in the capacitor by integrating $\frac{1}{2}\vec{D}\cdot\vec{E}$ over the relevant space.
   d. Compare your result from (a) with the result you find when using the expression of chapter 2 for electrostatic energy stored in a capacitor $\frac{1}{2}CV^2$ and an the expression for C for a cylindrical capacitor, i.e. $C = \frac{2\pi\varepsilon_o}{\ln(b/a)}l$

9. A sphere of a radius R, centered at the origin carries charge density

$$\rho(r,\theta) = 5\frac{R}{r^2}(R-2r)\sin(\theta)$$

Find the approximate potential for points on the z axis far from the sphere.